Empirical Eigenvectors of Sea-Level Pressure, Surface Temperature and Precipitation Complexes over North America

JOHN E. KUTZBACH

University of Wisconsin, Madison

(Manuscript received 16 February 1967, in revised form 3 July 1967)

ABSTRACT

The combined representation of fields of three climatic variables with empirical orthogonal functions, herein referred to as eigenvectors, is discussed. The eigenvectors are derived from measurements of monthly mean sea-level pressure, surface temperature and precipitation at 23 points in North America for 25 Januarys.

Selected eigenvectors of the individual climatic variables are presented; however, the major part of the paper is devoted to the presentation of eigenvectors consisting of combinations of three climatic variables. Empirical eigenvectors derived from fields of two or more meteorological variables have been used in statistical prediction models, but none of the studies to date displayed examples of these eigenvectors or discussed the internal consistency of the combined representations. In this paper it is shown that the structure of the covariances between the three climatic variables, as portrayed by the combined representations, is consistent with synoptic experience. This result illustrates that eigenvector representations derived from fields of several variables can be of considerable descriptive or diagnostic value.

1. Introduction

Representations of two-dimensional fields of meteorological variables with empirical orthogonal functions have been applied to a variety of problems since the 1950’s. These functions, herein referred to as eigenvectors, have certain advantages over the conventional orthogonal representations (Fourier series, Tschebycheff polynomials, etc.). For example, it can be shown via the calculus of variations that the eigenvector representation is optimum in the sense that maximum variance may be accounted for by choosing in order the eigenvectors associated with the largest eigenvalues of the appropriate covariance matrix. In addition, various investigators have reported that it is possible to provide physical explanations for the shapes of the horizontal patterns described by certain of the eigenvectors.

Most applications of this technique to meteorological problems have been in the realm of statistical prediction. In such cases, the orthogonality of the eigenvectors and the coefficients associated with them insures independent predictors, a desirable feature of statistical prediction equations. Lorenz (1956), Gilman (1957), and White et al. (1958) have conducted statistical studies using as predictors the coefficients associated with eigenvectors constructed from either surface temperature or sea-level pressure fields. Sellers (1957) has used an eigenvector representation of the 500-mb height field in a dynamical numerical prediction model.

Freiberger and Grenander (1965) have stressed the need for systematic empirical investigations of the structure of the covariances of the meteorological fields, including the first eigenvectors of the covariance matrices of relevant meteorological variables. Some work along these lines has been reported. For example, several investigators have used eigenvector representations to describe objectively the characteristics of fields or departure fields of individual meteorological variables. Grimmr (1963) has displayed eigenvectors of surface temperature over Europe; Borishekov (1966) has presented eigenvectors of surface temperature, incoming solar radiation and 500-mb heights for the hemisphere north of 50N; and Yudin (1966) has derived eigenvectors from pressure fields in connection with studies of major atmospheric oscillations.

The studies mentioned above have considered the representation of fields of individual meteorological variables. As pointed out by Lorenz (1956), eigenvector representations can be extended to include the combined representation of fields of several variables. Eigenvectors derived from horizontal fields of two or more meteorological variables have been used in statistical prediction models (Kelly, 1958; Aubert et al., 1959; Glaahn, 1962). However, none of these studies displayed examples of the eigenvectors or discussed the internal consistency of the combined representations.

In this study, the representation of two-dimensional fields composed of three climatic variables with empirically determined eigenvectors is discussed. The

1 Applications of this technique to the representation of vertical distributions [see, for example, Obukhov (1960) or Holmström (1963)] or time variations of individual meteorological variables [see, for example, Stidd (1967)] are not reviewed here.
eigenvectors are derived from measurements of monthly mean sea-level pressure, surface temperature and precipitation at 23 points in North America for 25 Januarys. These climatic variables were chosen because the interrelations between their mean fields are familiar to synoptic meteorologists and climatologists. It will be shown that the structure of the covariances between the three variables, as portrayed by the first several combined eigenvector representations, is consistent with synoptic experience. This result makes it possible to assess the usefulness of combined eigenvector representations in descriptive or diagnostic studies.

The mathematical procedure is outlined in Section 2. Section 3, the major section of this paper, contains illustrations of eigenvectors composed of various combinations of the three climatic variables and an examination of their synoptic consistency. A discussion of certain limitations and possible extensions of this study is presented in Section 4.

2. Procedure

The problem of representing fields of meteorological variables in terms of eigenvectors may be formulated in a number of different ways [see, for example, Lorenz (1956), Loeve (1963), and Freiberger and Grenander (1965)]. The approach followed here most closely resembles that of the latter and is chosen to emphasize the descriptive features of this representation.

Let \( f_n \) be an \( M \)-component vector representing the \( n \)-th observation, \( n = 1 \) to \( N \), of \( M \) variables and let \( F \) be an \( M \) by \( N \) matrix whose \( m \)-th column is the observation vector \( f_m \). The \( m \)-th observation of the \( m \)-th variable is denoted \( f_{mn} \). One wants to determine which vector \( e \) has the highest resemblance to all the observation vectors \( f \) simultaneously, where resemblance is measured with the squared and normalized inner product between a vector \( f \) and the vector \( e \). Thus, averaging across all the \( f \), one seeks to maximize the quantity

\[
(e'F)^2 N^{-1} = e' e,
\]

which is equivalent to maximizing

\[
e'RF
\]

subject to the condition

\[
e'e = 1,
\]

where \( R \) is an \( M \) by \( M \) symmetric matrix whose \( r_{ij} \)th element is given by

\[
r_{ij} = N^{-1} \sum_{n=1}^{N} f_{in} f_{jn},
\]

or

\[
R = N^{-1} (FF'),
\]

The prime denotes the transpose.

The maximization of (1) subject to the condition (2) is a variational problem which leads to the equation

\[
Re = \lambda e.
\]

The vector \( e \) and the parameter \( \lambda \) (which enters as a Lagrange multiplier) are now recognized as an eigenvector and associated eigenvalue of the symmetric matrix \( R \) and can be found by standard techniques. In fact, one obtains not one vector but a series \( e_i, i = 1, \ldots, M \), associated with the \( M \) eigenvalues of \( R \). It can be shown that the \( e_i \) are orthogonal and that the \( \lambda_i \) are real and positive.

Thus, writing (4) for all eigenvectors,

\[
R = \lambda E L \lambda,
\]

where \( E \) is an \( M \) by \( M \) orthogonal matrix whose columns are the eigenvectors \( e_1, \ldots, e_M \). Note that

\[
E' E = I.
\]

The matrix \( L \) is an \( M \) by \( M \) diagonal matrix whose \( i \)-th diagonal element \( \lambda_i \) is the eigenvalue associated with \( e_i \), the \( i \)-th column of \( E \). In the following, it is assumed that the elements of \( L \) and the columns of \( E \) have been arranged such that \( e_1 \) is associated with \( \lambda_1 \), the largest eigenvalue of \( R \), \( e_2 \) is associated with \( \lambda_2 \), the next largest eigenvalue of \( R \), and so forth. Combining (3) and (5), and noting from (6) that for orthogonal matrices the transpose is equal to the inverse, one obtains

\[
E' FE = L N. \quad (7)
\]

Defining

\[
C = F E\quad (8)
\]

where \( C \) is an \( M \) by \( N \) matrix, it follows that

\[
F = E C, \quad (9)
\]

and

\[
f_n = \sum_{i=1}^{M} c_{in} e_i, \quad n = 1, \ldots, N. \quad (10)
\]

From (10), it is clear that the observation vector \( f_n \), representing the \( n \)-th observation of the \( M \) variables, can be expressed as a linear combination of the \( M \) eigenvectors. The \( c_{in} \)-th element of \( C \) will be referred to as the coefficient associated with the \( i \)-th eigenvector for the \( n \)-th observation. From (8), it can be shown that \( c_{in} \) represents the projection of the \( n \)-th observation vector on the \( i \)-th eigenvector. The coefficients \( c_{in} \) play the same role in (10) as the coefficients in, for example, a Fourier series representation. The \( N \) observations refer to \( N \) different times and, therefore, the elements in the \( n \)-th row of \( C \) represent time variations of the coefficient associated with the \( i \)-th eigenvector.

Substituting (8) in (7), one obtains

\[
C C' = L N. \quad (11)
\]

That is, the row vectors of \( C \) are orthogonal. Thus, not only the eigenvectors but also the coefficients associated with them are orthogonal. The requirement of a com-
pletely orthogonal system, (6) and (11), and a linear transform (9), is sometimes used to derive (5).

It can be shown that the eigenvectors and their associated eigenvalues represent solutions of a maximization of "explained variance" problem. If the \( M \) variables are represented by departures from their sample means, the magnitude of the \( i \)th eigenvalue represents the amount of variance explained by the \( i \)th eigenvector [see (11) and (8)]. Thus, maximum variance is explained by choosing in order the eigenvectors associated with the largest eigenvalues of \( \mathbf{R} \). Since the trace of \( \mathbf{R} \) must be equal to the trace of \( \mathbf{L} \), the fraction of the total variance \( V_k \), explained by the \( k \) largest eigenvalues of \( \mathbf{R} \), can be obtained from

\[
V_k = \frac{\sum_{i=1}^{k} \lambda_i}{\sum_{i=1}^{M} \lambda_i}.
\]

In most applications of this technique, it is found that a large portion of the variance can be accounted for by retaining only the first few eigenvectors. In that case, the first few terms of (10) only approximate the observation vectors. If \( \mathbf{f}_n^* \) is defined as the approximation to the \( n \)th observation vector, then

\[
\mathbf{f}_n^* = \sum_{i=1}^{M^*} c_{in} \mathbf{e}_i, \quad n = 1, \cdots, N; \quad M^* < M.
\]

It will be helpful to review the preceding development in terms of the following geometric model before proceeding to a description of the results. The observation vectors \( \mathbf{f}_n \) may be regarded as a set of \( N \) vectors in an \( M \)-dimensional space where \( M \) is the total number of variables included in the representation. The \( n \)th vector is drawn from the origin to the point in \( M \)-space which specifies the value of the \( M \) variables for the \( n \)th observation. In the cases presented in Section 3, climatic variables will be defined at \( K \) locations in North America. Therefore, \( M \), the number of variables, will equal \( K \), \( 2K \) or \( 3K \) depending upon the number of climatic variables included (one, two or three, respectively). The \( N \) observation vectors will in general not be orthogonal. They will tend to "cluster" into a small number of groups if there are a small number of distinctly different distributions of the \( M \) variables. Using the technique described above, an orthogonal coordinate system consisting of \( M \) eigenvectors is obtained in which the first eigenvector is oriented to maximize the sum of squares of the projections of the observation vectors upon it. The second eigenvector is oriented to maximize the remaining sum of squares with the additional condition that it must be orthogonal to the first, and so on. Thus, eigenvectors other than the first may not pass directly through "clusters" of the observation vectors. This explains why linear combinations of several eigenvectors may be needed to represent an observation vector. The dimensions of the orthogonal vector space can normally be reduced since a large fraction of the total variance is explained in most cases by a small fraction of the \( M \) eigenvectors. For this reason, linear combinations of the eigenvectors in the reduced vector space approximate the original observation vectors.

3. Illustration of patterns

In order to illustrate the application of this technique to fields of several variables, the covariance structure of three common climatic variables (monthly mean sea-level pressure, surface temperature and precipitation) specified at a network of points on the North American continent for the Januaries of 1941–1965 will be discussed. The computer program available to calculate the eigenvectors and eigenvalues of \( \mathbf{R} \) was limited to 70 by 70 matrices. In order to incorporate the three variables at each point, it was necessary to restrict the network to 23 points (Fig. 1). The selected points are near the geographic centers of 23 regions of approximately equal area. The limits of these regions were chosen to avoid the crossing of obvious geographic boundaries or known climatic boundaries. Data from 2–5 climatological stations have been averaged within each region to obtain the basic data used in this study. The \( M \) variables are normalized, i.e., they have zero means and unit variances.

In general, if the \( M \) variables are represented by their actual values, departures from means, or normalized

![Fig. 1](image-url). The boundaries and approximate geographic centers of 46 regions. The analyses presented in subsequent figures are based on the network of 23 consecutively numbered points. Data from 2–5 stations were averaged within each region.
departs from means, the symmetric matrix $\mathbf{R}$ will be a cross-product matrix, covariance matrix, or correlation matrix, and the first eigenvector will have highest resemblance to the observed fields, departure fields, or normalized departure fields, respectively. The use of normalized variables assures that each variable at each point in the field is of equal importance in determining the form of the representation. In addition, normalized departure fields of temperature and precipitation correspond more closely to the climatological classifications of normal, above normal, much above normal, etc., than the departure fields themselves. This will aid in the interpretation of the local significance of particular features of the eigenvector patterns.

The mean map and standard deviation map for the 25 Januarys, 1941–1965, are presented in Fig. 2. The eigenvector patterns described below are derived from the 25 fields of observed normalized departures from the mean map. The 25 fields form the 25 columns (observation vectors) of the matrix $\mathbf{F}$. The observation vectors, and hence the eigenvectors, have respectively 23, 46 or 69 components if one, two or three climatic variables are specified at the 23 points. All of the eigenvector patterns are not shown. Only enough are reproduced to support the major points which are made. The following notation is used: $\mathbf{P}$ denotes eigenvectors containing only pressure variables, $\mathbf{T}$ temperature, $\mathbf{R}$ precipitation, $\mathbf{PT}$ pressure and temperature, and $\mathbf{PTRY}$ pressure, temperature and precipitation variables.

The first eigenvectors of $\mathbf{P}$, $\mathbf{T}$ and $\mathbf{R}$ are shown in Fig. 3. These patterns were obtained by plotting the 23 components of the eigenvectors at the 23 points shown in Fig. 1 and performing a hand analysis. Note that the isolines are nondimensional since the eigenvectors were derived from normalized departure fields. Each eigenvector actually defines two departure patterns because the coefficients associated with each eigenvector may be positive or negative [see Eq. (10)]. For example, if the sign of $c_{1,1}$ in (10) is positive (negative) in the $n$th January, the first eigenvector of $\mathbf{P}$ (Fig. 3a) depicts a large area of positive (negative) pressure departures over most of North America with a small area of negative (positive) departures in the southwestern portion of the United States. For the sake of brevity, the parenthetic descriptions for the cases of negative coefficients will be omitted in the following. Of course, the forms of the eigenvectors are influenced by the choice of stations, the particular years and months from which the data was taken, and the spatial extent of the network. This point may be illustrated by comparing the forms of the temperature eigenvectors computed in this study with those computed by Gilman (1957). His temperature functions were computed from mean monthly temperatures at a 30-station network over the United States for the winter months of 1900–
1939. His first temperature function closely resembles the United States portion of the second eigenvector of $T$ (Fig. 3d). His second temperature function most closely resembles the United States portion of the first eigenvector of $T$ (Fig. 3b), although his function describes east-west contrasts while the one shown here describes northwest-southeast contrasts.

Attention will now be focused on the combined representation of fields of several variables. The first eigenvector of $PT$ (a 46-component vector) and the first eigenvector of $PTR$ (a 69-component vector) are shown in Fig. 4. In the combined eigenvector representations, the isolines of pressure, temperature and precipitation are indicated by solid lines, dashed lines and
dashed-dotted lines, respectively. Regions of maxima or minima in the temperature patterns are identified by stippling or hatching. In comparing Fig. 4 with Fig. 3, one observes a close resemblance between the combined eigenvector patterns ($e_1$ of $PT$ and $PTR$) and the separate eigenvector patterns ($e_1$ of $P$, $T$ and $R$). There are, however, differences. For example, $e_1$ of $P$ is characterized by a broad area of positive pressure departures with maxima in the southern Canadian plains and the northern Middle West while the pressure portions of the departure patterns of $e_1$ of $PT$ and $e_1$ of $PTR$ are characterized by narrower ridges of positive pressure departures with maxima in Alaska and along the eastern coast of the United States. In fact, there are cols along the United States-Canadian border in the pressure departure patterns of $e_1$ of $PT$ and $e_1$ of $PTR$, whereas $e_1$ of $P$ displayed two small maxima in this area. Since the observed normalized departure fields of these three variables are not necessarily uniquely related, one expects differences between the combined and separate patterns. These differences are even more apparent in certain of the eigenvector patterns not shown. In general, the magnitudes of the departures are attenuated in the combined patterns.

The synoptic consistency of the departure patterns of the climatic variables in the combined eigenvector representations can be assessed in a qualitative fashion since the general relationships between observed surface temperature and precipitation departures and departures from the "normal" sea-level pressure distribution are well known [see Namias (1953) for a detailed review]. For example, inspecting the form of $e_1$ of $PT$ (Fig. 4a), it will be noted that the large region of negative departures in the temperature pattern covering much of the northern and western United States and western portions of Canada coincides with areas of more northerly than "normal" or less westerly than "normal" winds (as inferred from the gradients in the pressure departure pattern). The region of positive temperature departures in the southern and eastern part of the United States coincides with areas of more southerly than "normal" winds. The pressure and temperature components of $e_1$ of $PTR$ (Fig. 4b) exhibit the same general relationships. In addition, the area of the largest positive precipitation departures in the central portion of the United States coincides with the region of increased north-south temperature gradient. The area of the largest negative precipitation departures along the coast of British Columbia coincides with the area of less westerly than "normal" winds.

The second through the fifth eigenvectors of $PTR$ are presented in Fig. 5. As discussed above with regard to $e_1$ of $PTR$, the interrelationships among the pressure, temperature and precipitation departure patterns of these eigenvectors are also generally consistent with synoptic experience. For example, $e_2$ of $PTR$ is characterized by positive pressure departures over the central United States. There are negative temperature departures and negative precipitation departures over much of the central and eastern United States. These
areas coincide with regions of more northerly than "normal" winds. Temperatures are near their January means over much of the remainder of the map. There are positive precipitation departures in the southwestern United States and negative precipitation departures along the west coast. Note that the first of these regions coincides with an area of more southerly than "normal" winds, while the second coincides with a region of more easterly than "normal" winds. In the far north, temperature and precipitation departures are
small, but generally positive, in regions of more westerly or northwesterly than "normal" flow.

The pressure portion of $e_0$ of $\text{PTR}$ (Fig. 5) resembles a ridge-trough-ridge pattern. Over the United States, there are generally negative temperature departures to the west of the trough in the pressure departure pattern and positive temperature departures in and to the east of the same trough. The departure patterns of eigenvectors beyond the fifth exhibit characteristic space scales approaching the characteristic distance between points in the network. This trend is already apparent in the fourth and, especially, in the fifth eigenvector.

Although "noise" is present in all the eigenvectors, the eigenvectors associated with the smaller eigenvalues have more noise relative to their explained variance. The term noise is here understood to include variability confined to small space scales, purely local effects and observational errors.

4. Discussion

The major portion of Section 3 was devoted to the illustration of eigenvectors derived from normalized departure fields of several meteorological variables. It was shown, at least qualitatively, that the interrelations between the departure patterns of monthly mean sea-level pressure, surface temperature and precipitation, as portrayed by the first several eigenvectors, are consistent with synoptic experience.

Certain quantitative assessments of the results of Section 3 are discussed below. First, the efficiency of the combined eigenvector representations is examined. Second, several factors which limit the usefulness of these particular patterns are described. Third, the year-to-year variations in the magnitude and sign of the coefficients associated with the first several eigenvectors are examined.

1) The cumulative variance explained by the eigenvectors associated with the largest eigenvalues of $\mathbf{R}$ for each of the several variable combinations are presented in Table 1. It will be noted that the combined representations ($P$ or $\text{PTR}$) are more efficient than the separate representations ($P$, $T$, or $P$, $T$ and $R$) in the sense that the number of eigenvectors of $P$ or $\text{PTR}$ needed to explain a fixed amount of variance is always less than the total number of eigenvectors of $P$, $T$, or $P$, $T$ and $R$ required to reach the same level. For example, six eigenvectors of $P$ explain over 80% of the total variance whereas eight (four eigenvectors of $P$ and four of $T$) are needed to explain the same amount of variance when the fields of pressure and temperature are considered separately. Defined in this manner, efficiency is a function of the intercorrelation between the fields of the variables. It should be noted that there is a fundamental difference in approach between empirical prognostic and empirical diagnostic studies involving the use of eigenvectors. In the former, one tries to avoid the use of highly correlated variables, while in the latter their use may be highly desirable. For a given set of $M$ variables, the number of eigenvectors required to explain a specified portion of the total variance is inversely related to the degree of intercorrelation between the $M$ variables.

2) At least four factors limit the interpretability of the patterns presented in Section 3. First, the number of observations is small. If more Januarys were to be included, the forms and the relative order of the eigenvectors, particularly those associated with smaller eigenvalues, would be subject to change. A test of this sort was performed on a subset consisting of 15 of the 25 Januarys used in the present study. The first three eigenvectors of $\text{PTR}$ closely resembled those shown here, although the first two were reversed in order of importance. The higher order patterns showed significant changes. Second, the 23-point network allows examination of only the largest features of the departure patterns. A denser network of points would aid in resolving the smaller scales of variability.

Third, although the $M$ variables are normalized and therefore have equal variance, the distribution of explained variance using only, say, the first five eigenvectors of $\text{PTR}$ is not uniform from meteorological variable to meteorological variable or from point to point. These five eigenvectors explain approximately 70% of the total variance (see Table 1). Examining now the variable-to-variable differences, the first five eigenvectors of $\text{PTR}$ explain 77% of the pressure variance, 80% of the temperature variance, and 51% of the precipitation variance. These percentages roughly parallel the percentages of the total variance explained re-

### Table 1. Summary of the number of climatic variables and the total number of variables used in various models (top); and the cumulative per cent of total variance $V_k$ explained by the eigenvectors associated with the $k$ largest eigenvalues of their respective correlation matrices, computed from Eq. (12) (bottom).

<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>$P$</th>
<th>$T$</th>
<th>$R$</th>
<th>$PT$</th>
<th>$\text{PTR}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of climatic variables specified at each point $J$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Number of points on map $K$</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>Total number of variables $M$, $M = J$ times $K$</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>46</td>
<td>69</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$k$</th>
<th>Cumulative per cent of total variance $V_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36.2</td>
</tr>
<tr>
<td>2</td>
<td>58.7</td>
</tr>
<tr>
<td>3</td>
<td>73.0</td>
</tr>
<tr>
<td>4</td>
<td>79.6</td>
</tr>
<tr>
<td>5</td>
<td>85.6</td>
</tr>
<tr>
<td>6</td>
<td>89.8</td>
</tr>
<tr>
<td>7</td>
<td>93.5</td>
</tr>
<tr>
<td>8</td>
<td>97.1</td>
</tr>
<tr>
<td>9</td>
<td>99.0</td>
</tr>
<tr>
<td>10</td>
<td>99.5</td>
</tr>
</tbody>
</table>


spectively by the first four eigenvectors of $P$, $T$ and $R$. The low percentage of explained precipitation variance partially reflects the greater variability of January precipitation fields as compared to pressure and temperature fields. It is also true that the relationship between mean sea-level pressure and mean precipitation fields is less clear than the relation between mean sea-level pressure and mean surface temperature fields (Namias, 1953). Perhaps a variable such as within-month pressure variance would be more strongly related to mean monthly precipitation than the mean pressure. Next, considering the point-to-point variations, it is found that the variance explained by the first five eigenvectors of $PTR$ is higher in the interior of the network (11 points) than around the boundaries (12 points). In the interior region the first five eigenvectors of $PTR$ explain respectively 84%, 83% and 57%, and in the boundary region 70%, 75% and 45% of the pressure, temperature and precipitation variance. Averaging the three variables, the first five eigenvectors of $PTR$ explain 75% of the interior variance and 64% of the boundary variance. Sellers (1957) suggested that the boundary effect could be minimized by placing the borders of the network in regions of minimum variance. This, of course, would be of no help if normalized variables were used.

Finally, with respect to the use of normalized variables, it should be noted that the eigenvectors obtained in Section 3 differ from those that would be obtained from non-normalized departure fields since the meteorological variable to meteorological variable and point to point variances are not equal (see the standard deviation map, Fig. 2b). Some weighting procedure such as normalization is necessary in combined eigenvector representations to prevent the first several eigenvectors from being dominated by the meteorological variables with largest variance. However, normalization is not the only possible weighting procedure. Lorenz (1956) suggested that the different meteorological variables be weighted so that their average variances would be equal while retaining the real point-to-point differences in variance. The computations described in Section 3 were repeated using the properly weighted departure fields of the three climatic variables rather than the normalized departure fields to form the matrix $R$. The proper weighting was accomplished by multiplying all temperature departures by 1.215 and all precipitation departures by 0.109. The eigenvectors obtained in this manner, denoted by $\hat{\mathbf{e}}$, were compared with those obtained from the normalized variables. The general features of the patterns of the first several $\hat{\mathbf{e}}$ and $\mathbf{e}$ were identical. However, the magnitudes of the departures of the $\hat{\mathbf{e}}$ were frequently greater than those of the $\mathbf{e}$ at points where the climatic variables had large variance. Thus, the first several $\hat{\mathbf{e}}$ resemble departure fields rather than normalized departure fields. The eigenvectors $\hat{\mathbf{e}}$ permit an accurate interpretation of gradients in the pressure

departure patterns in terms of departures from "normal" wind. For example, in comparing Fig. 6 with Fig. 4, one finds strengthened gradients of pressure departure in the northwestern portions of the maps of $\mathbf{e}_1$ of $PT$ and $\hat{\mathbf{e}}_1$ of $PTR$. Similarly, there is a concentration of maximum temperature departures over the Canadian plains (Fig. 6a and 6b) and a localization of the major precipitation departures along the coast of British Columbia and in the Middle West (Fig. 6b). These areas correspond to regions of increased variance for the respective climatic variables. The eigenvectors derived from the non-normalized departure fields were slightly more efficient than those derived from the normalized departure fields. Thus, for various reasons, the use of non-normalized variables may be preferable to the procedure followed in Section 3.

3) It was shown in Section 3 that the departure patterns of the first several eigenvectors of $PTR$ exhibit realistic interrelations between the three climatic variables. However, it has not been shown that these patterns resemble observed normalized departure fields. Since the first eigenvector was chosen to have the highest resemblance to the observed normalized departure fields (see discussion of mathematical procedure in Section 2), this eigenvector can be expected to approximate the normalized departure field of particular Januarys within the 25-yr sample. The higher order eigenvectors, however, may be artifacts introduced by the orthogonality constraint. This possibility will now be examined.

In general, the observed normalized departure field for a particular January is not completely specified by a single eigenvector but is approximated by a linear combination of several. This fact is illustrated by a plot of the time variations of the coefficients associated with the first five eigenvectors of $PTR$ (Fig. 7). However, some Januarys show a close correspondence between a particular eigenvector of $PTR$ and the observed normalized field. The normalized departure fields of the Januarys of 1950 and 1958 closely resemble the first eigenvector of $PTR$. This can be seen by comparing the magnitudes of $c_{1,10}$ and $c_{1,18}$ with the magnitudes of the remaining $c_{1,15}$ and $c_{1,18}$ in Fig. 7. In addition, the second, third and fourth eigenvectors of $PTR$ each resemble one observed normalized departure field: namely, $\mathbf{e}_5$, January 1953; $\mathbf{e}_5$, January 1956; and $\mathbf{e}_5$, January 1941. None of the higher order eigenvectors of $PTR$ approximates an observed normalized departure field.

An objective indication of the number of eigenvectors needed to approximate the large-scale features of the observed normalized departure fields for particular Januarys is presented in Table 2. The tabulated values are the ratios of the sum of squares of the single largest, or two largest, coefficients to the sum of squares of the coefficients associated with the first five eigenvectors (labelled $\mathbf{e}_1$ and $\mathbf{e}_2$, respectively). Since eigenvectors beyond the fifth were dominated by small-scale or local
Fig. 6. a) First eigenvector of sea-level pressure and surface temperature ($\theta_1$ of $PT$), derived from non-normalized data; and b) first eigenvector of sea-level pressure, surface temperature and precipitation ($\theta_1$ of $PTR$), derived from non-normalized data. See note in legend of Fig. 4.

Fig. 7. Time variations of the coefficients associated with the first five eigenvectors of $PTR$ for 25 Januaries, 1941–1965. The notation is the same as in Eq. (10).
Table 2. Objective indication of the number of eigenvectors needed to approximate the observed normalized departure fields for particular Januarys. Left: the squares of the coefficients associated with the first five eigenvectors of P'TR. The largest and the second largest values for each January are denoted by the small superscripts, 1 and 2, respectively. Right: the sum of squares of the coefficients of the first five eigenvectors, denoted by SS; the ratio of the single largest value of \( c_{1n}^2 \) to SS, denoted by \( \beta_1 \); and the ratio of the sum of the two largest values of \( c_{1n}^2 \) to SS, denoted by \( \beta_2 \).

<table>
<thead>
<tr>
<th>January of</th>
<th>( n )</th>
<th>( c_{1n}^2 )</th>
<th>( c_{2n}^2 )</th>
<th>( c_{3n}^2 )</th>
<th>( c_{4n}^2 )</th>
<th>( c_{5n}^2 )</th>
<th>SS</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1941</td>
<td>1</td>
<td>0.00</td>
<td>0.50</td>
<td>4.92¹</td>
<td>20.28¹</td>
<td>0.68</td>
<td>26.38</td>
<td>0.77</td>
<td>0.96</td>
</tr>
<tr>
<td>1942</td>
<td>2</td>
<td>5.41</td>
<td>3.09</td>
<td>1.16¹</td>
<td>9.17¹</td>
<td>3.85</td>
<td>32.98</td>
<td>0.35</td>
<td>0.63</td>
</tr>
<tr>
<td>1943</td>
<td>3</td>
<td>5.53¹</td>
<td>0.56</td>
<td>3.80¹</td>
<td>2.92</td>
<td>0.60</td>
<td>13.50</td>
<td>0.41</td>
<td>0.69</td>
</tr>
<tr>
<td>1944</td>
<td>4</td>
<td>15.24¹</td>
<td>0.50</td>
<td>13.79</td>
<td>15.54¹</td>
<td>0.64</td>
<td>45.71</td>
<td>0.34</td>
<td>0.67</td>
</tr>
<tr>
<td>1945</td>
<td>5</td>
<td>6.22¹</td>
<td>3.61</td>
<td>1.21</td>
<td>4.46¹</td>
<td>0.54</td>
<td>16.04</td>
<td>0.39</td>
<td>0.67</td>
</tr>
<tr>
<td>1946</td>
<td>6</td>
<td>0.73</td>
<td>4.37¹</td>
<td>16.84¹</td>
<td>2.85</td>
<td>1.00</td>
<td>25.79</td>
<td>0.65</td>
<td>0.82</td>
</tr>
<tr>
<td>1947</td>
<td>7</td>
<td>0.20</td>
<td>19.90¹</td>
<td>11.22¹</td>
<td>3.30</td>
<td>2.40</td>
<td>37.02</td>
<td>0.54</td>
<td>0.84</td>
</tr>
<tr>
<td>1948</td>
<td>8</td>
<td>8.00¹</td>
<td>12.53¹</td>
<td>1.72</td>
<td>6.55</td>
<td>3.72</td>
<td>32.61</td>
<td>0.38</td>
<td>0.63</td>
</tr>
<tr>
<td>1949</td>
<td>9</td>
<td>47.65¹</td>
<td>1.52</td>
<td>24.68</td>
<td>0.28</td>
<td>35.12¹</td>
<td>109.25</td>
<td>0.44</td>
<td>0.76</td>
</tr>
<tr>
<td>1950</td>
<td>10</td>
<td>146.00¹</td>
<td>8.29¹</td>
<td>0.92</td>
<td>0.00</td>
<td>0.02</td>
<td>155.23</td>
<td>0.94</td>
<td>0.99</td>
</tr>
<tr>
<td>1951</td>
<td>11</td>
<td>2.43¹</td>
<td>1.96</td>
<td>3.76¹</td>
<td>0.34</td>
<td>0.08</td>
<td>8.67</td>
<td>0.43</td>
<td>0.71</td>
</tr>
<tr>
<td>1952</td>
<td>12</td>
<td>12.65¹</td>
<td>12.24¹</td>
<td>2.67</td>
<td>2.55</td>
<td>1.51</td>
<td>31.62</td>
<td>0.40</td>
<td>0.79</td>
</tr>
<tr>
<td>1953</td>
<td>13</td>
<td>2.39</td>
<td>48.22¹</td>
<td>4.53¹</td>
<td>0.63</td>
<td>3.45</td>
<td>59.22</td>
<td>0.81</td>
<td>0.89</td>
</tr>
<tr>
<td>1954</td>
<td>14</td>
<td>16.29¹</td>
<td>2.82</td>
<td>10.16¹</td>
<td>1.31</td>
<td>1.30</td>
<td>31.88</td>
<td>0.51</td>
<td>0.83</td>
</tr>
<tr>
<td>1955</td>
<td>15</td>
<td>8.59</td>
<td>35.40¹</td>
<td>0.00</td>
<td>3.52</td>
<td>10.10¹</td>
<td>57.61</td>
<td>0.61</td>
<td>0.79</td>
</tr>
<tr>
<td>1956</td>
<td>16</td>
<td>0.10</td>
<td>0.00</td>
<td>60.00¹</td>
<td>2.43</td>
<td>7.83¹</td>
<td>70.39</td>
<td>0.85</td>
<td>0.96</td>
</tr>
<tr>
<td>1957</td>
<td>17</td>
<td>32.41¹</td>
<td>17.42¹</td>
<td>1.19</td>
<td>0.70</td>
<td>4.53</td>
<td>56.32</td>
<td>0.58</td>
<td>0.89</td>
</tr>
<tr>
<td>1958</td>
<td>18</td>
<td>51.27¹</td>
<td>4.17¹</td>
<td>2.02</td>
<td>0.00</td>
<td>4.06</td>
<td>61.52</td>
<td>0.83</td>
<td>0.90</td>
</tr>
<tr>
<td>1959</td>
<td>19</td>
<td>0.07</td>
<td>1.75</td>
<td>17.51¹</td>
<td>3.64</td>
<td>5.52¹</td>
<td>28.49</td>
<td>0.61</td>
<td>0.81</td>
</tr>
<tr>
<td>1960</td>
<td>20</td>
<td>1.22</td>
<td>0.38</td>
<td>0.18</td>
<td>1.38¹</td>
<td>3.96¹</td>
<td>7.12</td>
<td>0.56</td>
<td>0.75</td>
</tr>
<tr>
<td>1961</td>
<td>21</td>
<td>15.75¹</td>
<td>17.38¹</td>
<td>0.03</td>
<td>0.29</td>
<td>4.49</td>
<td>37.94</td>
<td>0.46</td>
<td>0.87</td>
</tr>
<tr>
<td>1962</td>
<td>22</td>
<td>0.73</td>
<td>20.45¹</td>
<td>6.05</td>
<td>11.49¹</td>
<td>3.13</td>
<td>41.85</td>
<td>0.49</td>
<td>0.76</td>
</tr>
<tr>
<td>1963</td>
<td>23</td>
<td>0.50</td>
<td>58.20¹</td>
<td>0.59</td>
<td>31.08¹</td>
<td>3.10</td>
<td>93.47</td>
<td>0.62</td>
<td>0.96</td>
</tr>
<tr>
<td>1964</td>
<td>24</td>
<td>22.09¹</td>
<td>28.33¹</td>
<td>13.68</td>
<td>11.91¹</td>
<td>0.01</td>
<td>76.02</td>
<td>0.37</td>
<td>0.66</td>
</tr>
<tr>
<td>1965</td>
<td>25</td>
<td>1.63</td>
<td>3.53¹</td>
<td>1.76</td>
<td>3.67¹</td>
<td>0.79</td>
<td>11.38</td>
<td>0.32</td>
<td>0.63</td>
</tr>
</tbody>
</table>

Effects, these ratios may be interpreted as fractions of the large-scale variations explained by the eigenvectors associated with the largest coefficients. Note that \( \beta_1 \) exceeds 0.7 five times while \( \beta_2 \) exceeds 0.7 in 18 of the 25 cases. The observed normalized departure field of January 1963 provides an example of a month described by a linear combination of two eigenvectors (\( e_2 \) and \( e_4 \)). In certain cases, Januaries with large coefficients associated with two of the eigenvectors were months with two distinctly different departure fields, each resembling one of the eigenvectors. The January of 1961 [see the description of the weather of this month by Green (1961)], for which the coefficients associated with the first and second eigenvectors were large, provides a good example of this. Results of this sort are probably fortuitous since monthly mean data, which average the effects of many regimes, were used in this study. If longer observational series consisting of shorter period means (5–10 days) were used, one would expect to obtain a larger number of eigenvectors whose forms would more closely resemble the departures from the mean state associated with particular circulation regimes.

5. Summary

The main object of this paper has been to describe a heretofore unemphasized application of eigenvector representations, namely, the study of the interrelationships between fields of several meteorological variables. It was shown that the patterns of the first five eigenvectors of sea-level pressure, surface temperature and precipitation depict realistically the covariance structure of the fields of these climatic variables. This encourages the application of this technique to descriptive or diagnostic studies in which the interrelationships between fields of several variables are not clearly understood. At the same time, this study illustrates a new approach to the quantitative and objective description of climatic regimes through the use of combined eigenvector representations.

Acknowledgments. This research was supported by the Atmospheric Science Division, National Science Foundation, Grants GP-444 and GP-5572X.

REFERENCES


