Introduction to Radiation

Planetary Energy Balances
Radiant energy propagates as waves characterized by wavelength or frequency (number of waves per sec)
Covers very wide range of wavelengths
Also behaves like particles

- Energy comes in discrete packets
  - Photons
  - Energy in a photon is inversely proportional to wavelength
    - \( E = h \nu = hc/\lambda \)
    - \( h \) is Planck’s constant \((6.626 \times 10^{-34} \text{ J s})\), \( \nu \) is frequency (Hz)
    - Blue light with \( \nu = 6 \times 10^{14} \) Hz has quantum energy \( 4 \times 10^{-19} \) J

- Planck function gives energy emitted for black body
  - \( B(\nu, T) = (2h\nu^3/c^2)/(\exp(h\nu/kT)-1) \)
    - Boltzmann constant \( k \) \((1.38 \times 10^{-23} \text{ J/K})\) converts temperature in K to energy
    - Ratio \( h\nu/kT \) essentially quantum energy compared to average molecular energy
    - For terrestrial temperatures and visible light, ratio is around 50-100, so almost no energy emitted
    - Ratio reaches 1 for frequencies around \( 6 \times 10^{12} \) Hz = 48 \( \mu \text{m} \) (far IR)
The quantum vs the classical world

- Let $u = \frac{\hbar \nu}{kT}$ be non-dimensional energy ratio
  - Rewrite Planck’s law
    - $B(\nu, T) = \frac{(2k^3T^3/h^2c^2) \times (u^3/(e^u-1))}{(e^u-1)}$
  - Ratio reaches 1 for frequencies around $6 \times 10^{12}$ Hz = 48 μm (far IR) at terrestrial temperatures, $10^{14}$ Hz = 2.5 μm (near IR) at solar temperatures
  - Classical limit: $u \ll 1$
    - $e^u - 1 \approx u$
    - $u^3/(e^u-1) \approx u^2$
    - $B \approx 2kTv^2/c^2$
    - Independent of $h$
    - Energy emitted increases quadratically with frequency
  - Quantum limit: $u \gg 1$
    - $u^3/(e^u-1) \approx u^3e^{-u}$
    - Decays somewhat more slowly than exponential at higher frequencies
  - Faster decay at high frequency end makes emission curves asymmetrical with low frequency tail stretching farther from peak
Electromagnetic Spectrum

- Dense materials emit over range of wavelengths
  - Blackbody radiation – emits maximum predicted by Planck’s law at all wavelengths
- Total energy emitted increases rapidly with increasing temperature
  - Stefan-Boltzmann law integrates Planck’s law over frequency (wavelength): \( E = \sigma T^4 \)
- Wavelength of maximum emission inversely (frequency directly) related to temperature
  - Wien’s Law: \( \lambda_{\text{max}} = \frac{2900}{T} \)
Solar and Terrestrial Radiation

• Sun’s surface temperature about 5800K
  – $E = 6.4 \times 10^7 \text{ W/m}^2$
  – $\lambda_{\text{max}} = 0.48 \mu\text{m} (1 \mu\text{m} = 10^{-6} \text{ m})$
  – Almost all energy at wavelengths less than 4 microns
    • 6.7% ultraviolet
    • 46.8% visible
    • 46.5% near infrared

• Earth’s surface temperature about 300K
  – $E = 460 \text{ W/m}^2$
  – $\lambda_{\text{max}} = 9.8 \mu\text{m}$
  – Almost all energy at wavelengths greater than 4 $\mu\text{m}$
Virtually no overlap between spectra
Solar radiation at top of atmosphere

- **Solar luminosity** – total energy emitted per second
  - Solar energy flux = $6.4 \times 10^7$ W/m$^2$
  - Radius of sun = $6.96 \times 10^8$ m
  - Luminosity = $3.8 \times 10^{26}$ W (J/s)

- **Intensity of energy flux decreases with square of distance from sun**
  - Surface area of sphere illuminated by energy increases with square of distance (radius)
  - Earth’s mean distance from sun = $1.5 \times 10^{11}$ m
  - Mean flux at top of atm (solar constant) = $1367$ W/m$^2$
Planetary energy balance

• Earth receives radiant energy from sun
  – Solar constant $X$ earth’s cross-sectional area
  – $E = E_s \left( \frac{r_s}{r_0} \right)^2 \pi a^2$
  – $E_s$ is solar flux, $r_s$ is sun’s radius, $r_0$ is radius of earth’s orbit, $a$ is earth’s radius

• Some is reflected away by clouds, light surfaces, aerosols, etc.
  – Earth’s planetary albedo (portion reflected) is about 0.3

• Earth must emit an equal amount of energy to space
  – Energy flux $X$ earth’s surface area
  – $E = \sigma T^4 4\pi a^2$
Planetary energy balance

• Absorbed solar radiation = emitted terrestrial radiation
  – \((1-\alpha) \ E_s \ (r_s/r_0)^2 \ \pi a^2 = \sigma T^4 \ 4\pi a^2\)
  – \(\alpha\) is planetary albedo
  – \((1-\alpha) \ E_s \ (r_s/r_0)^2 = 4\sigma T^4\)

• Can solve for \(T\) to get emission equilibrium temperature
  – Value for earth is 255K
  – Difference between 255K and surface temperature of 288K reflects strength of greenhouse effect